(7 pages) Reg. No. : ....

Code No. : 30937 E **Sub. Code : FCMA 12**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

First Semester

Mathematics–Core

DIFFERENTIAL CALCULUS

(For those who joined in July 2024 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. $\frac{d}{dx}\left(\cos h^{-1} x\right) =$ ________.

(a) $\frac{1}{\sqrt{x^2 - 1}}$ (b) $\frac{-1}{\sqrt{x^2 - 1}}$

(c) $\frac{1}{\sqrt{x^2 + 1}}$ (d) $\frac{-1}{\sqrt{x^2 + 1}}$

2. $\frac{d^5}{dx^5}\left(e^{2x}\right)$ is ______.

(a) $16e^{2x}$

(b) $32e^{2x}$

(c) $2^4 e^{2x}$

(d) None

3. $\frac{\partial^2 f}{\partial x \partial y}$ is denoted by.

(a) f_{xy}

(b) f_{yx}

(c) f_{xx}

(d) f_{yy}

4. If $x = r\cos\theta$, then $\frac{\partial x}{\partial \theta}$ is.

(a) $r\sin\theta$

(b) $r\cos\theta$

(c) $-r\cos\theta$

(d) $-r\sin\theta$

5. $f(x, y, z)$ is a homogenous function of 2nd degree, $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} =$ ______.

(a) $2f$

(b) $2^2 f$

(c) $2^3 f$

(d) f

6. If $AC - B^2 > 0$ and $A > 0$ (or $B > 0$) then $f(a,b)$ is.

(a) maximum

(b) minimum

(c) neither (a) or (b)

(d) (a) or (b)

7. The Parametric form of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is.

(a) $(a \cos\theta, a \sin\theta)$

(b) $(a \sec\theta, b \tan\theta)$

(c) $(a \cos\theta, b \sin\theta)$

(d) $(a \cos\theta, b \cos n\theta)$

8. For $f(x, y, \alpha) = 0$, $\frac{dy}{dx}$ at $(\xi, \eta) =$ __________.

(a) $\frac{\partial f / \partial \xi}{\partial f / \partial \eta}$ (b) $\frac{\partial f / \partial \eta}{\partial f / \partial \xi}$

(c) $\frac{-\partial f / \partial \xi}{\partial f / \partial \eta}$ (d) $\frac{-\partial f / \partial \eta}{\partial f / \partial \xi}$

9. For a circle, radius of curvature is equal to.

(a) radius of the circle

(b) square of the radius

(c) square root of the radius

(d) reciprocal of radius

10. On a given curve, at any point curvature is.

(a) $\frac{ds}{d\psi}$ (b) $\frac{d\psi}{ds}$

(c) $\frac{ds}{dx}$ (d) $\frac{ds}{dy}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that

$$\frac{d^n}{dx^n}\left(\frac{1}{(ax+b)^2}\right) = \frac{(-1)^n a^n (n+1)!}{(ax+b)^{n+2}}.$$

Or

(b) Find y_n, where $y = \frac{3}{(x+1)(2x-1)}$.

12. (a) If $u = \log(\tan x \quad \tan y + \tan z)$ prove $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$.

Or

(b) Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where $y = \frac{1-x}{x}$.

13. (a) If $u = \sin^{-1}\left[\frac{x^2 + y^2}{x + y}\right]$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$.

Or

(b) If $z = f(x, y)$ and $x = r \cos\theta$, $y = r \sin\theta$, prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

14. (a) Find the envelope of the family of straight lines $x \cos\alpha + y \sin\alpha = a \sec\alpha$, α being the parameter.

Or

(b) Find the envelope of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ subject to $a^2+b^2+c^2$, where C is constant.

15. (a) Find P at $x=c$ on $xy=c^2$.

Or

(b) Find the radius of curvature of the curve $r^n = a^n \cos n\theta$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) If $y=\left(\sin^{-1}x\right)^2$ prove that $\left(1-x^2\right)y_{n+2}-(2n+1)x\ y_{n+1}-n^2y_n=0$.

Or

(b) Find the n^{th} derivative of $x^2\log 3x$.

17. (a) If $u=\log\left(x^2+y^2+z^2\right)$, prove

$$x\frac{\partial^2 u}{\partial y\partial z}=y\frac{\partial^2 u}{\partial z\partial x}=z\frac{\partial^2 u}{\partial x\partial y}.$$

Or

(b) If $z = (1 - 2xy + y^2)^{-\frac{1}{2}}$ prove $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = y^2 z^3$.

18. (a) Find the maximum and minimum of $x^2 + y^2 + 6x + 12$.

Or

(b) In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$.

19. (a) Find the envelope of the family of straight lines $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$.

Or

(b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ subject to $a^n + b^n = c^n$, C is constant.

20. (a) Show that the evolute of the cycloid $X = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

Or

(b) Find the radius of curvature at $(a\cos^3\theta, a\sin^3\theta)$ on $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.
